STUDY OF DYNAMICS OF CONTINUOUS FUNCTIONS ON TOPOLOGICAL SPACES

(Summary Report)

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In Mathematics, Topological Dynamics is a branch of the theory of dynamical systems in which qualitative, asymptotic properties of dynamical systems are studied from the viewpoint of general topology.

A dynamical system is a concept in Mathematics where a fixed rule describes the time dependence of a point in a geometrical space. Mathematical models that describe the swinging of a pendulum, the flow of water in a pipe and the number of fish each spring time in a lake are some examples.

At any given time, a dynamical system has a “state” given by a set of real numbers (a vector) that can be represented by a point in an appropriate “state space” (geometrical manifold). Small changes in the state of the system create small changes in the numbers. The evolution rule of the dynamical system is a fixed rule that describes what future state follows from the current state. The rule is deterministic; in other words, for a given time interval, only one future state follows from the current state.

The central object of study in topological dynamics is a topological dynamic system that is a topological space together with a continuous transformation. The origin of topological dynamics lie in the study of asymptotical properties of trajectories of systems of autonomous ordinary differential equations. A structure Theorem for minimal distal flows proved by Hillel Furstenburg in the early 1960's inspired much work on classification of minimal flows. A lot of research work in the 1970's and 1980's have been conducted in the subject.

Applications

Dynamical systems theory attempts to understand or atleast describe the changes over time that occur in physical and artificial"systems". Examples of such systems include the solar system, weather, sugar dissolving in a cup of coffee, the stockmarket, the formation of traffic jams, behavior of the decimal digits of the square root of 2 etc.

Many areas of Biology, Physics, Economics and applied Mathematics involves a detailed analysis of systems like these, based on the particular laws governing their change. These laws in turn are derived from a suitable theory : Newtonian mechanics, fluid dynamics, Mathematical Economics and so on.

All these models can be unified conceptually in the mathematical notion of a dynamical system, which consists of two parts: the phase space and the dynamics. The dynamics is a rule that transforms one point in the phase space representing the state of the system "now", in to another point representing the state of the system, one time unit later. In mathematical language, the dynamics is a function mapping world states in to world states.

For example, if we are studying planetary motion, then a world state might consists of the location and velocities of all planets and stars in some neighbourhood of the solar system, and the dynamics would be derived from the laws of gravity, which, given the position and masses of the planets, determine the forces acting on them. Once an initial world state is chosen, the dynamics determines the world state at all future times.

Abstract dynamics is the study of dynamical systems based on the descriptions explained above and discarding most specialized informations about the system or the origin of dynamics.

A very general problem in abstract dynamics is to understand when two systems are the "same". The dynamics of dissolving a drop of red ink in a cup of water is essentially the same as the process of a drop of black ink. In this example, if we run time backwards, we get a different dynamics than running it forward, because when time goes forward, the ink becomes more and more spread out and dissolved where as if time goes backward, it starts outdissolved and becomes more and more concentrated in a drop. This observation is the essence of the second law of thermodynamics.

Chaotic dynamical systems exhibit sufficient “randomness” and “unpredictability”. Examples of chaotic dynamics include many physical systems- weather, socio-economic systems, stock market etc. Infact, most natural systems are chaotic.

An invariant of a dynamical system is some quantity we can measure which repeats after a certain period of time. Periodicity of a system is an example. The weather is approximately periodic with a period of 365 days.

There are also many interesting applications of dynamics theory in Mathematics- especially in combinatorics, number theory and the fundamentals of information theory.

The idea of a “lift” given in chapter 3, has applications in Algebraic Topology. (F.H.Croom, Basic concepts of Algebraic topology)

In this project work, I have tried to make a brief study on dynamical systems, especially dynamics of continuous functions on the topological spaces R and C. As mentioned above ,dynamical systems have lot of applications in practical life. Some of them are discussed in this project.